

# A Flexible Multi-body Dynamic Model for Analyzing the Hysteretic Characteristics and the Dynamic Stress of a Taper Leaf Spring

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This paper proposes a modeling technique which is able to not only reliably and easily represent the hysteretic characteristics but also analyze the dynamic stress of a taper leaf spring. The flexible multi-body dynamic model of the taper leaf spring is developed by interfacing the finite element model and computation model of the taper leaf spring. Rigid dummy parts are attached at the places where a finite element leaf model is in contact with an adjacent one in order to apply contact model. Friction is defined in the contact model to represent the hysteretic phenomenon of the taper leaf spring. The test of the taper leaf spring is conducted for the validation of the reliability of the flexible multi-body dynamic model of the taper leaf spring developed in this paper. The test is started at an unloaded state with the excitation amplitude of 1~2 mm/sec and frequency of 132 mm. First, the simulation is conducted with the same condition as the test. Then, the simulations are conducted with various amplitudes in a loaded state. The hysteretic diagram from the test is compared with the ones from the simulation for the validation of the reliability of the model. The dynamic stress analysis of the taper leaf spring is also conducted with the developed flexible multi-body dynamic model under a dynamic loading condition.

**Key Words :** Multi-leaf Spring, Taper Leaf Spring, Hysteretic Characteristics,  
Dynamic Spring Rate, Dynamic Stress, Finite Element Model,  
Flexible Multi-body Dynamic Model, Contact Model

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## 1. Introduction

A multi-leaf spring is mainly used in the suspension system of a commercial vehicle like a truck or bus. Since the multi-leaf spring is also working as a structural member, the suspension system of a truck or bus does not require a link-

age system. This makes it possible to lower the installation height of the multi-leaf spring. However, a ride comfort of a vehicle with the multi-leaf spring can deteriorate because the dynamic spring rate is significantly increased by decreasing the excitation amplitude due to the hysteretic characteristics of the multi-leaf spring. In addition, the multi-leaf spring can induce noise because of interleaf friction. Generally, it is considered that the multi-leaf spring has reached in the uppermost limit for improving the dynamic characteristics required for developing a high performance truck.

A taper leaf spring has two to four leaves, which is much less leaves than the multi-leaf spring. If the thickness and shape of the taper leaf spring are designed well, the taper leaf spring can improve the ride comfort due to interleaf friction area being reduced more than 3 times, and can reduce the noise induced from interleaf friction. In addition, it can reduce the weight more than 30% since it has less leaves than the multi-leaf spring. Thus, studies have been performed for utilizing the taper leaf spring instead of the multi-leaf spring in recent years (Kim and Lim, 1996; SAE HD788, 1990).

A study on the dynamic characteristics and interleaf friction of a leaf spring was performed through a test (Yamamoto, 1995). Studying a leaf spring through a test is not easy to perform since it is difficult to make an apparatus for the test, and it takes a lot of time to conduct the test. These difficulties hinder conducting tests of a leaf spring with a variety of conditions. In recent years the dynamic characteristics of a leaf spring has been studied by developing a computational model of the leaf spring. A leaf spring was modeled with four links including a shackle, joints, rotational springs, and bushing elements (Antoun, 1986). The model was used to study a kinematical trajectory of an axle during jounce and rebound. A computational model was developed for studying the effect of interleaf friction on the dynamic characteristics of a leaf spring (Song, 1993). However, the study did not consider the hysteretic characteristics of the leaf spring. The dynamic spring rate defined by the diagonal slope of the hysteretic

diagram of a leaf spring is considerably varied as the working displacement varies due to the hysteretic characteristics of the leaf spring. Therefore, the hysteretic characteristics of a leaf spring significantly affects the dynamic characteristics of a vehicle. A modeling technique representing the leaf spring with links is widely used for including the hysteretic characteristics of the leaf spring in the model (Judd, 1990). However, the modeling technique requires tuning of many variables, and what is worse, it requires the values of the longitudinal and lateral stiffness of the leaf spring, which are very difficult to decide. Developing a modeling technique of a leaf spring, which is easy to model, and represents the hysteretic characteristics reliably, is needed. In addition, it has been demanded that the development of a computational model of a leaf spring, which can be used for the dynamic analysis for optimally design, and then used for the fatigue life estimation of the leaf spring.

This paper proposes a modeling technique for not only reliably and easily representing the hysteretic characteristics but also analyzing the dynamic stress of a taper leaf spring. The flexible multi-body dynamic model of the taper leaf spring is developed by interfacing the finite element model and computational model of the taper leaf spring. The reliability of the developed flexible multi-body dynamic model of the taper leaf spring is verified by comparing the hysteretic diagram from the simulation performed with the developed model with the one from the test. A dynamic stress analysis of the taper leaf spring is also performed with the developed model to show the possibility of the usage of the developed model in the optimal design and the fatigue life analysis of the taper leaf spring.

## **2. Flexible Multi-body Dynamic Modeling of a Taper Leaf Spring**

This paper develops a flexible multi-body dynamic model of a taper leaf spring used in the suspension system of a large truck. Fig. 1 is a schematic diagram of the taper leaf spring used in this paper before the taper leaf spring is installed.

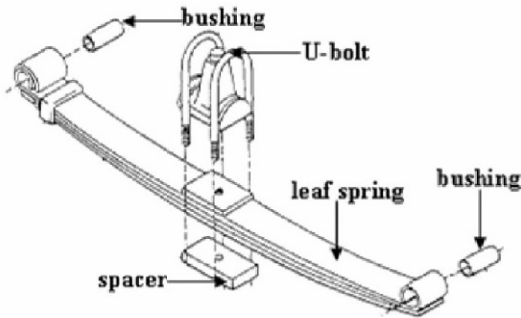


Fig. 1 Schematic diagram of a taper leaf spring

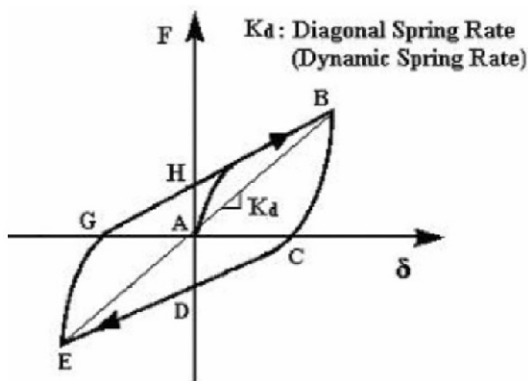


Fig. 2 Load-deflection diagram of a leaf spring

As it shows, one end of the taper leaf spring is connected to the frame with a bushing, and the other end of it is connected to the frame through a shackle. The shackle allows the rotational and translational motion of the taper leaf spring with respect to the frame while the taper leaf spring deforms elastically. This means that the taper leaf spring model to be developed in this paper must be able to represent not only the elastic deformation but also the translational motion of the taper leaf spring.

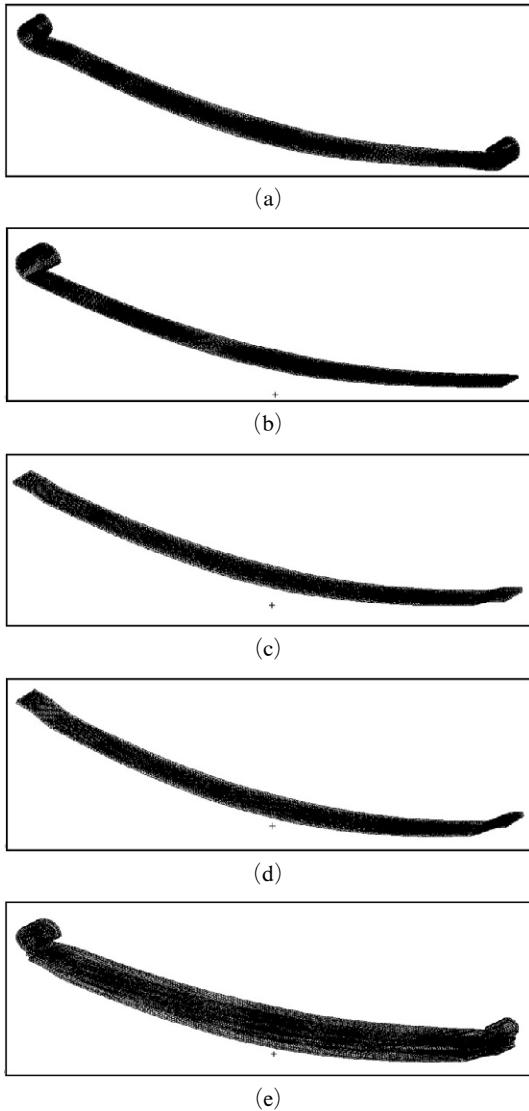
Figure 2 shows a load – deflection diagram of a leaf spring. The load-deflection diagram called a hysteresis diagram is formed as a load is being applied and released to the leaf spring. When the leaf spring is compressed and released according to an applied load, it deforms elastically. This elastic deformation of the leaf spring induces the relative motion of two adjacent leaves. The relative motion of two adjacent leaves induces interleaf friction. This interleaf friction makes two

transitional deflection regions “BC” and “EG” in the hysteresis diagram. The translational deflection regions determine the hysteresis characteristics of the leaf spring. A vehicle’s dynamic characteristics is significantly affected by the dynamic spring rate and damping of the leaf spring. The dynamic spring rate or diagonal spring rate is defined by the slope of the line made by diagonally connecting the two points “B” and “E” in the hysteresis diagram. The damping is determined by the area enclosed by the hysteresis diagram.

The solid model of the taper leaf spring was prepared with CAD software (Manual, 1998). Then, it was transferred to MSC.PATRAN (User’s manual, 1994) for finite element modeling of the taper leaf spring. Fig. 3 shows the finite element model of the taper leaf spring. The first leaf as shown in Fig. 3(a) was composed of 11,300 nodes, 8,000 hexagonal elements, and 5 MPCs (multi-point constraints). The second leaf as shown in Fig. 3(b) was composed of 13,011 nodes, 9,216 hexagonal elements, and 6 MPCs. The third leaf as shown in Fig. 3(c) was composed of 7,206 nodes, 4,776 hexagonal elements, and 6 MPCs. The fourth leaf as shown in Fig. 3(d) was composed of 7,158 nodes, 4,752 hexagonal elements, and 4 MPCs. Fig. 3(e) shows the finite element model of the taper leaf spring. It was made with 38,675 nodes, 26,744 hexagonal elements, and 21 MPCs. The density, modulus of elasticity, and Poisson’s ratio of the finite element model were set to 7,850 kg/m<sup>3</sup>, 200 GPa, and 0.3 respectively since the taper leaf spring was made with steel.

When the taper leaf spring deforms elastically, interleaf friction occurred because leaves always kept in contact with adjacent leaves at the middle and both ends of the taper leaf spring. This paper develops the computational model of the taper leaf spring using MSC.ADAMS (User’s manual, 1994). The computational model connects the finite element model of the taper leaf spring to the ground body with rotational and translational joints allowing for the rotational and translational motion, and to define contact between two adjacent leaves.

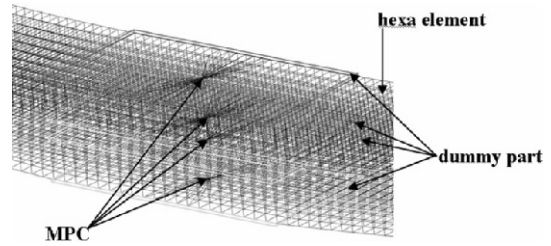
Contact between adjacent leaves is modeled with “contact model” in MSC.ADAMS. Contact



**Fig. 3** Finite element model of the taper leaf spring (a) 1<sup>st</sup> leaf, (b) 2<sup>nd</sup> leaf, (c) 3<sup>rd</sup> leaf, (d) 4<sup>th</sup> leaf, (e) taper leaf spring

model in MSC.ADAMS can not apply in contact between finite element models. Dummy parts were made of rigid bodies. They were attached at the places where the finite element leaf models were in contact with the adjacent ones. Therefore, contact in leaves occurred between dummy parts instead of the finite element models of the taper leaf spring.

An independent node was used for joining the finite element model to the rigid dummy part. The

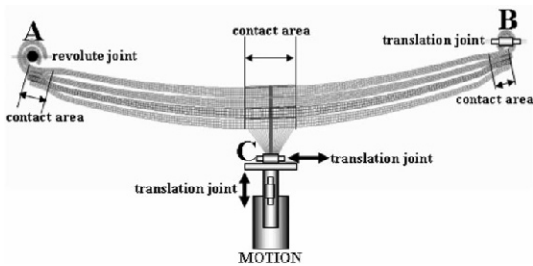


**Fig. 4** Contact model of the taper leaf spring

independent node was generated at the middle of the area where two adjacent leaves were in contact. The nodes placed on contact area were connected to the independent node with MPC elements. This made it possible to describe the rigid body motion of the nodes placed on contact area. Thus, it was possible that the relative displacement between two adjacent dummy parts affected finite elements placed on contact areas. Fig. 4 shows one of the places where two adjacent leaves were in contact in detail.

The hysteretic characteristics of the taper leaf spring was represented by contact model defined between two adjacent dummy parts. The values required in contact model are the stiffness, stiffness force exponent, static friction coefficient, and dynamic friction coefficient. Stiffness and stiffness force exponent were major factors which affected the dynamic spring rate of the taper leaf spring. Since metal and metal were in contact in the taper leaf spring, stiffness and stiffness force exponent were set to  $1.0 \times 10^5$  N/mm and 2 respectively. Friction between two adjacent leaves was considered as Coulomb friction. Static and dynamic friction coefficients were found through a trial and error process, and were set to 0.25 and 0.15 respectively.

In order to model the connection of the fore part of the taper leaf spring (part A in Fig. 5) with the frame, a dummy part was attached at the fore part of the first leaf with a fixed joint, and then the dummy part was joined to the frame with a revolute joint. In order to model the connection of the aft part of the taper leaf spring (part B in Fig. 5) with the frame, a dummy part was attached at the aft part of the first leaf with a fixed joint, and a dummy part was attached at the frame. Then, the dummy part attached at the first



**Fig. 5** Flexible multi-body dynamic model of the taper leaf spring

leaf spring was joined to the dummy part attached at the frame with a translational joint in order to allow the translational motion in fore and aft direction of the taper leaf spring. Friction was defined between the dummy parts joined with the translational joint. It was represented with a friction coefficient, and significantly affected the hysteretic characteristics of the taper leaf spring. Static and dynamic friction coefficients were found through a trial and error process, and were set to 0.4 and 0.3 respectively.

The middle of the taper leaf spring was connected with the axle. A load was transmitted to the taper leaf spring through the axle. The middle of the taper leaf spring moved not only in up and down direction but also in fore and aft direction as the load was being applied and released. Two translational joints were introduced to model this behavior at the middle section of the taper leaf spring. They were arranged along fore and aft, and up and down direction. To define the two translational joints, three dummy parts were generated. One dummy part was attached at the middle of the fourth leaf. The other two dummy parts were located under the dummy part attached at the middle of the fourth leaf spring, and attached at the ground part. One of the dummy parts attached at the ground part was used to define the translational joint allowing up and down direction movement. The other one attached at the ground part was used to define the translational joint allowing fore and aft direction movement. Up and down displacement was applied to the taper leaf spring instead of applying a load. It was applied at the translational joint used to allow the up and down movement.

An MSC.NASTRAN (User's manual, 1994) executable file was made with the finite element model of the taper leaf spring. A normal mode analysis was conducted with the executable file and MSC.NASTRAN/DMAP module. Node numbers, natural frequencies, mode shapes, etc. were acquired from normal mode analysis, and were saved in an output file. MSC.ADAMS/FLEX module was used to transform the output file to a modal neutral file (MNF) which can be interfaced with the MSC.ADAMS computational model of the taper leaf spring. The modal neutral file was composed of internal nodes and external nodes. Internal nodes were the nodes used to compose hexagonal elements for the finite element modeling of the taper leaf spring. External nodes were the nodes used to interface the finite element model with the MSC.ADAMS computational model of the taper leaf spring. The flexible multi-body dynamic model of the taper leaf spring was made through the above described process. Fig. 5 shows the flexible multi-body dynamic model of the taper leaf spring developed in this paper.

### 3. Validation of the Flexible Multi-body Dynamic Model of the Taper Leaf Spring

The test of the taper leaf spring was conducted for the validation of the reliability of the flexible multi-body dynamic model of the taper leaf spring developed in this paper. The taper leaf spring used for the test had a thickness of 24 mm, a width of 90 mm, and 4 leaves. Fig. 6 is a schematic diagram of the apparatus for the test. As Fig. 6 shows, a hydraulic actuator was used for applying a load to the taper leaf spring. The load was applied at the middle of the taper leaf spring since the middle of the taper leaf spring was connected to an axle. A U-bolt was used to bind the middle of the taper leaf spring. The test of the taper leaf spring was conducted without an initial deflection; that is, it was conducted from an unloaded state. The excitation speed of the test was 1~2 mm/sec, and the excitation amplitude was 132 mm.

The flexible multi-body dynamic model was

made to be the same status as the apparatus for the test. A simulation was conducted with the

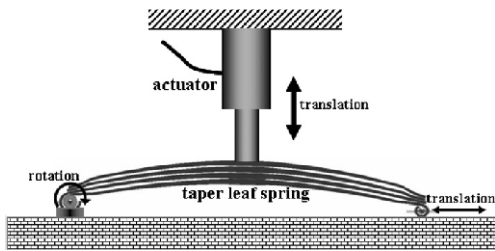


Fig. 6 Schematic diagram of an apparatus for the test of the taper leaf spring

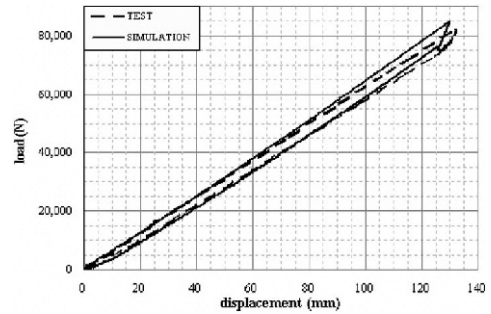


Fig. 7 Comparison of the hysteretic diagram of the taper leaf spring from the simulation and the one from the test

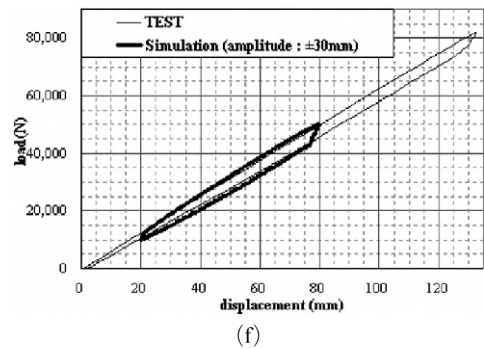
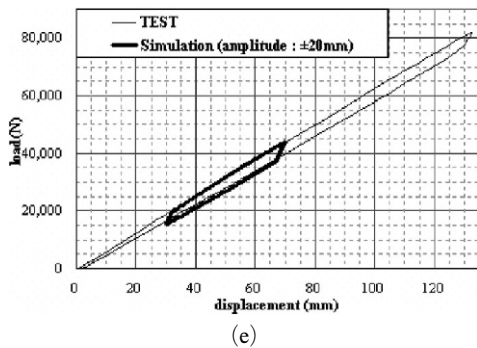
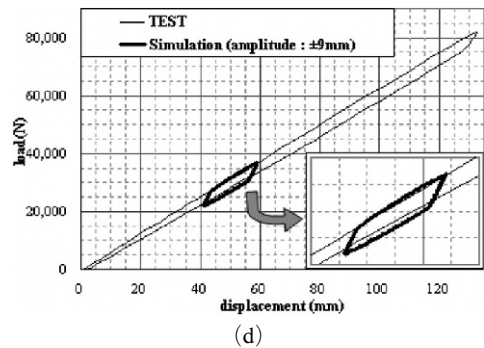
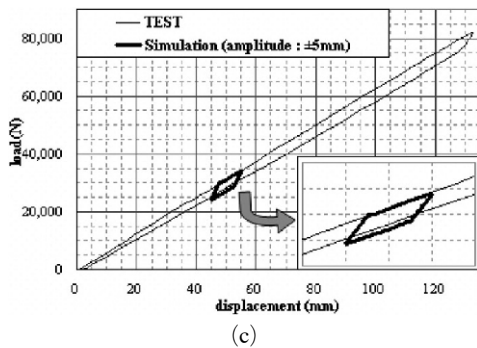
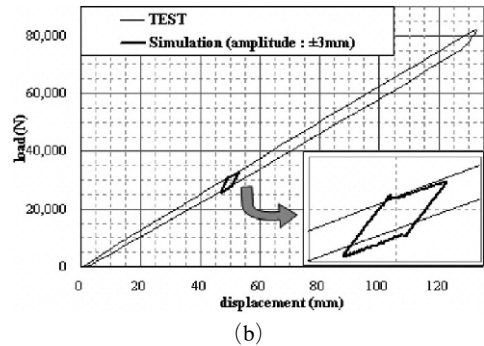
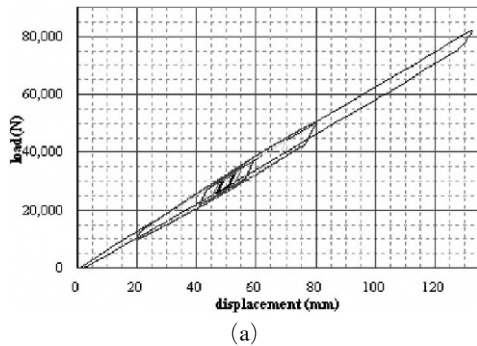


Fig. 8 Comparison of the simulation results with the test result in various excitation amplitudes (a) all amplitudes, (b)  $\pm 3$  mm, (c)  $\pm 5$  mm, (d)  $\pm 9$  mm, (e)  $\pm 20$  mm, and (f)  $\pm 30$  mm

developed flexible multi-body dynamic model of the taper leaf spring under the same condition as the test. Fig. 7 compares the hysteretic diagram of the taper leaf spring acquired from the simulation and the one acquired from the test. As it shows, the hysteretic diagram acquired from the simulation has a good correlation with the one from the test not only in the slope but also in the tendency of the diagram.

Since the excitation amplitude significantly affects the hysteretic diagram, but the excitation frequency affects the hysteretic diagram very little (Moon and Oh, 2004) simulations were conducted with various amplitudes regardless of the excitation frequency. Simulations were conducted with the excitation amplitudes of  $\pm 3$ ,  $\pm 5$ ,  $\pm 9$ ,  $\pm 20$ , and  $\pm 30$  mm. Simulations were conducted with an initial deflection of 50.03 mm corresponding to an axial load of 29,233.6 N. Fig. 8 compares the simulation results with the test result. As Fig. 8(b) shows, the hysteretic diagram from the simulation conducted with the small excitation amplitude of  $\pm 3$  mm has a little difference in the slope with the one from the test. However, in general, the simulation results have a good correlation with the test result not only in the slope but also in the tendency of the diagram.

Since the developed flexible multi-body dynamic model of the taper leaf spring can reliably represent the hysteretic characteristics, it will contribute on improving the reliability of the computational model of not only the taper leaf spring itself but also a commercial vehicle for the dynamic characteristics analysis.

#### 4. Dynamic Stress Analysis of the Taper Leaf Spring

A dynamic stress analysis of the taper leaf spring can be conducted with the developed flexible multi-body dynamic model since the flexible multi-body model represented the taper leaf spring with finite elements. In the dynamic stress analysis of the taper leaf spring, the excitation frequencies of 1 Hz, 3 Hz and 5 Hz, and the excitation amplitudes of  $\pm 3$  mm,  $\pm 5$  mm and  $\pm 9$  mm were considered as dynamic loading condi-

tions.

Figure 9 shows one of dynamic stress analysis results, and shows the dynamic stress distribution of the taper leaf spring. As anticipated, maximum stress occurred just beside the middle of the taper leaf spring since the middle of the taper leaf spring was bound by a U-bolt. Fig. 10 compares the maximum dynamic stresses occurred under various dynamic loading conditions. As Fig. 10 shows, the maximum dynamic stress increased as the excitation frequency or amplitude increased. The maximum dynamic stress was influenced more by the excitation amplitude than by the excitation frequency. In future, the flexible multi-body dynamic model of the taper leaf spring will be able to be used for the computation of the dynamic stress of the taper leaf spring under a real driving condition, and also used for extracting a loading history for the durability analysis of the taper leaf spring.

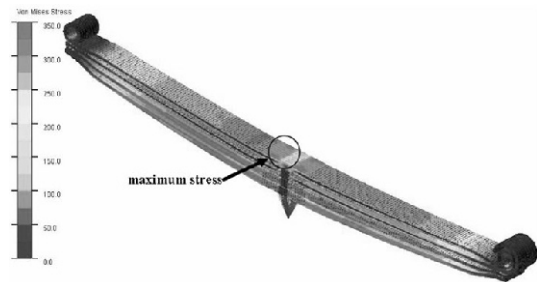


Fig. 9 Von-Mises stress distribution of the taper leaf spring subjected to a dynamic load

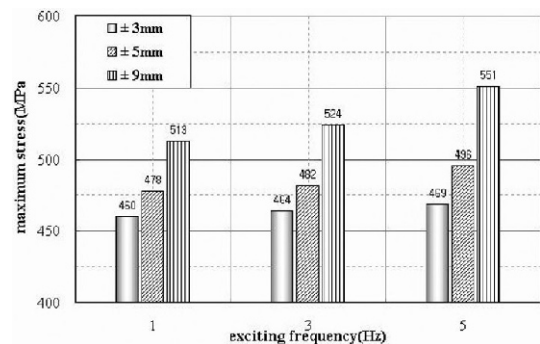


Fig. 10 Comparison of the maximum Von-Mises stresses of the taper leaf spring subjected to various dynamic loads

## 5. Conclusions

This paper developed the flexible multi-body dynamic model which can reliably represent the hysteretic characteristics and analyze the dynamic stress of the taper leaf spring. The flexible multi-body dynamic model was made by interfacing the finite element model and computational model of the taper leaf spring. The total finite element model of the taper leaf spring was composed of 38,675 nodes, 26,744 hexagonal elements, and 21 MPCs. Rigid dummy parts were attached at the places where the finite element leaf models were in contact with the adjacent ones in order to apply the contact model. Friction was defined in the contact model to represent the hysteretic characteristics of the taper leaf spring. The test of the taper leaf spring was conducted for the validation of the reliability of the flexible multi-body dynamic model of the taper leaf spring developed in this paper. The test was started at an unloaded state. The excitation amplitude and frequency of the test were 1~2 mm/sec and 132 mm respectively. The simulation was conducted with the same condition as the test. The hysteretic diagram from the simulation had a good correlation with the one from the test. The simulations were conducted with various amplitudes in a loaded state. The simulation results also had a good correlation with the test result even though the hysteretic diagram from the simulation conducted with the small excitation amplitude of  $\pm 3$  mm which showed little difference in the slope with the one from the test. The dynamic stress analysis of the taper leaf spring was conducted with the developed flexible multi-body dynamic model with the excitation frequencies of 1 Hz, 3 Hz and 5 Hz, and the excitation amplitudes of  $\pm 3$  mm,  $\pm 5$  mm and  $\pm 9$  mm. The maximum stress was occurred just beside the middle of the taper leaf spring. The maximum dynamic stress was increased as increasing the excitation frequency or amplitude. The maximum dynamic stress was influenced more by the excitation amplitude than by the excitation frequency. The flexible multi-body dynamic model of the taper leaf spring will be able

to contribute to improving the reliability of the computational model of a commercial vehicle for the dynamic characteristics analysis. In addition, it will be able to extract a loading history for the durability analysis of the taper leaf spring.

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